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Emergent quantum jumps in a nano-electro-mechanical system

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Abstract
We describe a nano-electro-mechanical system that exhibits the 'retroactive' quantum jumps discovered by Mabuchi and Wiseman (1998 *Phys. Rev. Lett.* **81** 4620). This system consists of a Cooper-pair box coupled to a nano-mechanical resonator, in which the latter is continuously monitored by a single-electron transistor or quantum point contact. Further, we show that these kinds of jumps, and the jumps that emerge in a continuous quantum non-demolition measurement, are one and the same phenomena. We also consider manipulating the jumps by applying feedback control to the Cooper-pair box.

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In 1998, Mabuchi and Wiseman discovered the existence of emergent jumps in a purely diffusive, measured cavity QED system, and termed the phenomena 'retroactive' quantum jumps [1]. Since that time, it has also been discovered that quantum jumps will emerge in a noisy system subjected to a continuous quantum non-demolition (QND) measurement. Examples are the continuous measurement of the energy of a two-level system, such as a Cooper-pair box (CPB) [2], and the measurement of the energy of a nano-mechanical resonator [3–8]. Our main purpose here is to show that the jumps discovered by Mabuchi and Wiseman, and the jumps in a QND measurement, are, in fact, the same phenomena having the same underlying origin. While there was no explicit QND measurement in the cavity QED scenario of Mabuchi and Wiseman, the 'retroactive' jumps are nevertheless due to a QND measurement that arises as a result of the interaction between the two subsystems. This is a QND measurement on one subsystem mediated through the other subsystem. These jumps may therefore be described as arising from an *indirect* QND measurement, rather than a direct one. Thus, if one interprets the cQED jumps of Mabuchi and Wiseman as retroactive, then the jumps induced by all QND measurements, whether on quantum or classical systems, are retroactive in the same sense.

In the process of our analysis will also do a number of other things. We first show how to design a nano-electro-mechanical system so that the jumps of the 'retroactive' type emerge in the diffusive motion. This system consists of a nano-mechanical resonator linearly coupled to a CPB [9], where the position of the resonator is continuously measured. This measurement can be implemented using a single-electron transistor (SET) [10] or a quantum point contact (QPC) [11]. The dynamics of this system are not identical to the cavity-QED system analyzed in [1], and this difference makes it easier to see the connection with the usual direct QND measurement. We also show that it is possible to effect a degree of control over the jumps by applying feedback control to the CPB.

If one places a CPB adjacent to a nano-resonator, and places a voltage on the nano-resonator, then the resonator feels a linear force from the CPB, and this depends upon which state the CPB is in. (That is, whether or not the CPB contains an excess Cooper-pair.) The Hamiltonian describing the dynamics of the coupled systems is [12]

$$H = \hbar[\omega_R a^\dagger a + \lambda\sigma_z(a + a^\dagger) + \omega_C\sigma_z + \omega_J\sigma_x]. \tag{1}$$

Here, a is the annihilation operator for the resonator, which has an angular frequency ω_R. The frequency corresponding to the charging energy of the CPB is ω_C, and the Josephson tunneling frequency is $\omega_J\sigma_x$. The tunneling term allows us to perform σ_x rotations using voltage pulses, but otherwise can be ignored since we set $\omega_C \gg \omega_J$. The interaction between the resonator and the CPB is the linear force that the resonator feels from the charge on the CBP, and the strength of this force determines λ. The expression for the above rate parameters in terms of the physical configuration of the resonator and CPB may be found, for example, in [12, 13].

As indicated above, the position of the resonator can be monitored continuously using an SET or QPC. We model this measurement process as an inefficient, but otherwise ideal, continuous position measurement [13–15]. The continuous stream of measurement results (often referred to as the measurement record) is $r(t)$, where $\mathrm{d}r = \langle x\rangle\,\mathrm{d}t + \mathrm{d}W/\sqrt{8\eta k}$ and $\mathrm{d}W$ is an increment of the Gaussian white noise [16]. Here, k is a measure of the rate at which information is extracted from the system, and which we will refer to as the measurement strength. The parameter η is called the efficiency of the measurement. The interaction of the system with its environment (including the SET) continually carries information away from the system (at a rate proportional to k), and η gives the fraction of this information which is actually collected by the observer. The resulting dynamics of the system density matrix, ρ, is given by the stochastic master equation (SME) [11, 17]:

$$\mathrm{d}\rho = (-\mathrm{i}/\hbar)[H,\rho]\,\mathrm{d}t - k[x,[x,\rho]]\,\mathrm{d}t + \sqrt{2\eta k}(x\rho + \rho x - 2\langle x\rangle\rho)\,\mathrm{d}W, \tag{2}$$

where H is given by equation (1) above and x is the position operator for the resonator. As such, k has units of $\mathrm{m}^{-2}\,\mathrm{s}^{-1}$, and so we define the corresponding dimensionless rate $\tilde{k} = k(\hbar/2m\omega)$.

We will consider two modifications of the above basic dynamics. The first is that we will modulate the interaction strength λ between the resonator and the CPB at the resonant frequency of the resonator, so that $\lambda = \lambda_0\cos(\omega_R t)$. This can be done by varying the voltage on the resonator. The result is to allow the CPB to drive the resonator at its resonant frequency, generating the maximum steady-state displacement of the resonator. The second modification is the application of a real-time feedback loop [13, 18] to damp the motion of the resonator. This means that the observer continually applies a force $F(t) = -\gamma\langle p(t)\rangle$ to the resonator, where $\langle p(t)\rangle = \mathrm{Tr}[p\rho(t)]$ is the observer's maximum likelihood estimate of the momentum of the oscillator at each time t. The result of this is to apply a (somewhat noisy) frictional damping force to the resonator.

We must also include in the dynamics the effects of temperature on the resonator. The resonator is in contact with a thermal bath that induces damping and injects noise into the resonator. Since the quality factor of the resonator is above 10^4, the thermal damping is much smaller than the damping that will be induced via the feedback loop, and as a result we simply subsume this damping into the feedback. The thermal noise can be taken into account by choosing an appropriate value for the efficiency η [13]. The noise introduced by the (low temperature) thermal bath is just as if a position detector was carrying information away at the rate $k_{\rm therm} = (m\omega_{\rm R}\Gamma)/(2\hbar)\coth(\hbar\omega_{\rm R})/(2k_B T)$ [13, 19], where Γ is the thermal damping rate and T is the temperature. To include the thermal noise, one therefore replaces k in equation (2) with $k_{\rm tot} = k + k_{\rm therm}$, and η with $\eta_{\rm tot} = (k/k_{\rm tot})\eta$, where η is the SET measurement efficiency.

The final thing to include is the environmental noise on the CPB. The kind of noise that is of interest to us is the noise which causes diffusion between the two energy eigenstates. It is this noise which, coupled with the system's dynamics, induces the quantum jumps. Thermal noise is of this type, and is usually modeled with the master equation $\dot{\rho} = \kappa\{(\xi+1)\mathcal{D}[\sigma_-]+\xi\mathcal{D}[\sigma_+]\}\rho$, where $\xi = 1/(e^{\hbar\omega_{\rm C}/(k_B T)} - 1)$ and $\mathcal{D}[a]\rho \equiv [a^\dagger a, \rho]_+ - 2a\rho a^\dagger$ for any operator a. However, the charging energy of a CPB is usually chosen so that $\hbar\omega_{\rm C}/(k_B T) \ll 1$. In this case the thermal noise will only cause the upper state to decay, rather than to generate diffusion between the two.

Nevertheless, there are other ways to induce the desired diffusion. One is to apply a stochastic sequence of pulses to a voltage gate, where the pulses bring the CPB to the degeneracy point. If the random pulse lengths are short compared to $\omega_{\rm J}$, then the Josephson tunneling term generates an evolution described by a Hamiltonian $\zeta(t)\sigma_x$, where $\zeta(t)$ is the white noise with autocorrelation $\langle\zeta(t)\zeta(t+\tau)\rangle = \kappa\delta(\tau)$. The result is the master equation $\dot{\rho} = \kappa\mathcal{D}[\sigma_x]\rho$. This master equation closely emulates the thermal master equation in the limit in which $\xi \gg 1$.[3] Third, one could instead increase the Josephson term. Due to the measurement dynamics to be discussed later, this should have the same effect as thermal noise.

In our numerical simulations, we choose to explicitly model the second noise source discussed above, that of stochastic driving proportional to σ_x. As a result, the full dynamics of the resonator-CPB system is

$$\begin{aligned} d\rho = (-i/\hbar)[H(t) - \gamma x\langle p\rangle, \rho]\,dt - k_{\rm tot}[x,[x,\rho]]\,dt \\ + \kappa\mathcal{D}[\sigma_x]\rho\,dt + \sqrt{2\eta_{\rm tot}k_{\rm tot}}(x\rho + \rho x - 2\langle x\rangle\rho)\,dW. \end{aligned} \tag{3}$$

Following the analysis of the cavity-QED system by Mabuchi and Wiseman, we now examine the steady state of the Hamiltonian $H(t)$ including the feedback damping, when the CPB is in either of its energy eigenstates. We will denote these eigenstates by $|\pm\rangle$—they correspond to the presence or absence of a Cooper-pair in the box. In these two cases, the resonator has the effective Hamiltonian

$$H_\pm = \hbar\omega_{\rm R}a^\dagger a - (\gamma\langle p\rangle \pm F\cos(\omega_{\rm R}t))x, \tag{4}$$

where $F = \lambda\sqrt{2\hbar m\omega_{\rm R}}$ is the maximum value of the driving force. Since the Hamiltonian is linear, the dynamics of the expectation values of x and p are simply those for the equivalent classical system, namely that of a driven, damped harmonic oscillator. As a result, the steady-state solution for $\langle x(t)\rangle$ is

$$\langle x(t)\rangle = \frac{F}{\gamma\omega_{\rm R}m}\cos(\phi + \omega t), \qquad \phi = \mp\frac{\pi}{2}, \tag{5}$$

where m is the mass of the resonator. The state of the resonator in each case is a (somewhat squeezed) Gaussian state. Thus, like the cavity-QED system in [1], our system has two stable orthogonal states, each of which is a product of a distinct state of the two-level system with a

[3] Making the approximation $\xi \gg 1$, the thermal noise term becomes $k\xi(2\rho - \sigma_x\rho\sigma_x - \sigma_y\rho\sigma_y)$. Since the system Hamiltonian is approximately symmetric in x and y, the effect on the system is equivalent to $2k\xi(\rho - \sigma_x\rho\sigma_x)$.

distinct Gaussian state of the resonator. In our case, it is the steady-state phase of the resonator, ϕ, that is tightly correlated with the orthogonal states of the CPB.

The measurement of the position of the oscillator continually provides the observer with information regarding the location of the oscillator in phase space, and thus about the phase of the oscillations. Since this phase is correlated with the eigenstates of the oscillator, the measurement will tend to continually collapse the state of the CPB to one of its eigenstates. Because of this it is only the two eigenstates that are stable against the measurement process, and this is the reason that the two steady states given by equation (5) are important—if the environmental noise is to induce jumps in the system, it will be between these two stable states.

We now simulate the full dynamics of the observed nano-electro-mechanical system, including the environmental noise. In specifying values for the system parameters, we will quote all rate constants in terms of the frequency of the resonator $f = \omega_R/(2\pi)$. We set the interaction strength $\lambda = 0.5f$ and the feedback damping rate $\gamma = 0.25f$, both of which are easily achievable [12, 13]. We set the SET measurement strength at $\tilde{k} = 0.01f$, which is also not difficult to achieve, certainly with f as high as 10 MHz [13]. We find from our simulations that relatively high efficiency ($\eta_{\text{tot}} \geqslant 0.7$) is required for the observer to effectively track the quantum jumps. This requires that the SET have high efficiency, and that $k_{\text{therm}} \ll k$. The question is still open as to whether such an efficiency can be reached with an SET [20], but it is estimated that a QPC should in theory be able to achieve efficiencies above $\eta = 0.8$.[4] Setting $Q = 10^5$, and using the parameters in [13], gives $k_{\text{therm}} \approx 5k$. Thus, a factor of 20 increase in k from that configuration would be required. While we would expect this to be possible, the overall efficiency requirement is the most challenging in the scenario. Finally, we choose the CPB noise strength to be $\kappa = 0.01f$.

In figure 1(*a*), we show the evolution of the phase of the resonator, $\theta(t)$, which we define by the relation $A(t)\,\mathrm{e}^{\mathrm{i}\theta(t)-\mathrm{i}\omega_R t} = \langle\tilde{x}(t)\rangle + \mathrm{i}\langle\tilde{p}(t)\rangle$. Here, $\tilde{x}$ is as defined above and $\tilde{p} = -\mathrm{i}(a - a^\dagger)$. We start the resonator in a coherent state with $\langle x\rangle = 3\Delta x$ and $\langle p\rangle = 0$, so that the initial phase is zero. We see that the phase quickly drifts to one of the two values $\pm\pi/2$, and from then on exhibits jumps in the motion between these values. In figure 1(*b*), we show the position of the resonator as a function of time. The amplitude of the position oscillations tends to reduce during phase flips, as expected.

One of the most interesting aspects of these quantum jumps is that they are an emergent phenomenon; the underlying dynamics does not contain jumps but consists purely of continuous diffusion. The authors of [1] explain this emergence by providing a detailed analysis of the interplay of the correlations produced by the Hamiltonian dynamics, the measurement-induced localization, and the diffusive noise. Their analysis is certainly not wrong. Our point here is that the mechanism that produces jumps in this case is the same mechanism that produces jumps in the case of a continuous QND measurement. This provides us with a somewhat simpler picture of the cause of the 'retroactive' jumps.

QND measurements are the measurements in which the observable being measured is not changed by the dynamics of the system (that is, the observable commutes with the Hamiltonian) [21]. As a result, once the measurement has projected the system onto a given eigenstate of the observable, it remains there throughout the remainder of the observation period. Now consider what happens when the observable is additionally subject to diffusion from the environmental noise. In the absence of the measurement, the noise will cause the observable to diffuse from one eigenstate to another, but in the presence of a sufficiently strong continuous QND measurement the dynamics is quite different. Consider what happens during a small time

[4] A N Korotkov, private communication.

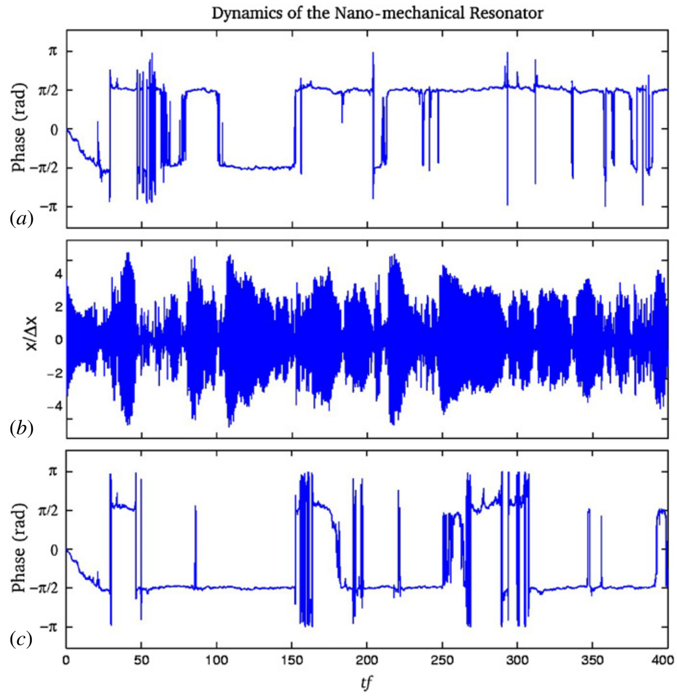

Figure 1. Here we plot the evolution of the nano-mechanical resonator under continual position measurement: (*a*) the phase of the resonator; (*b*) the mean position of the resonator; (*c*) the phase of the resonator when feedback control is applied to the Cooper-pair box. For (*a*) and (*b*) $\eta_{\rm tot} = 0.7$, and for (*c*) $\eta_{\rm tot} = 0.95$.

(This figure is in colour only in the electronic version)

interval when the system begins in one eigenstate. During the interval the diffusion will generate a small probability that the system is in an adjacent eigenstate. However, during the same interval, the measurement will collapse the system to one of the eigenstates, and with very high probability this will be the initial eigenstate, since the diffusion has only managed to generate a small probability for the other eigenstates during the short time interval. As a result, it is only unlikely events, in which the noise has a particularly large fluctuation, and the measurement conspires by returning a low-probability result, that will cause the system to transition from one eigenstate to the next. The result is the periods in which the system remains in a given eigenstate of the QND observable, interspersed by quantum jumps between the eigenstates. The jumps are *quantum* jumps since they only appear because the observable has a discrete spectrum. This qualitative picture has been confirmed by numerical simulations of a measurement of the energy of a quantum harmonic oscillator [3, 6], and the states of an electron in a coupled pair of quantum dots (often referred to as a 'charge qubit') [2].

The jumps discovered in [1], and those in the dynamics here, can now be seen to be caused by the same effect, except in that in these two cases the QND measurement is mediated through a second system. In our case, the QND observable is the energy of the CPB. The interaction between the resonator and the CPB causes the phase of the resonator to be tightly correlated with the energy eigenstates of the CPB, as shown in equation (5). In continually providing information about the phase of the resonator, the position measurement necessarily provides information about the energy of the CPB, generating a QND measurement and resulting in quantum jumps. Interestingly, it is not necessary to perform a QND measurement on the mediating system (the resonator) to generate the jumps; position is not a QND observable for the resonator.

We now consider the use of feedback control to stabilize the system against the jumps. In doing so, we reveal a fundamental limitation of feedback control in this context—the Hamiltonian feedback cannot stop the system from jumping. The reason for this is quite simple—as discussed above, the jumps from $|-\rangle$ to $|+\rangle$, for example, are due to the small probability for $|+\rangle$ continually generated by the noise process. To generate this probability, the noise process merely reduces the length of the Bloch vector. Since the cause of the jumps is purely this reduction (along with the QND measurement), and since the Hamiltonian evolution, feedback or otherwise, can rotate the Bloch vector but cannot change its length, the feedback is powerless to prevent the jumping. However, feedback can be used to *increase* the rate of jumps from either state, thereby altering the relative time that the system spends in the two states. Ultimately this could be used to ensure that the system spends almost all its time in one of the states, creating an effective stability.

We implement this procedure by applying a feedback Hamiltonian of the form $H = \mu(n_x(t)\sigma_x + n_z(t)\sigma_z)$ to the CPB, where $n_x^2 + n_z^2 = 1$, so that μ determines the feedback strength. In each time step $\Delta t = 1/(2500f)$, we choose this Hamiltonian so as to rotate the system towards the state $|-\rangle$. We set $\mu = 200$, and find that $\eta_{\text{tot}} = 0.95$ is required to obtain a significant effect. We plot the resulting evolution in figure 1(*c*), for the same noise realization as before. This shows that the phase is now more stable at $-\pi/2$ than $\pi/2$.

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